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Development of a simple dynamic model for fast fracture research

J. P. DEAR
Department of Mechanical Engineering, Imperial College of Science and Technology, Exhibition Road, London SW7 2BX, UK

The aim of this study was to devise a simple fracture model, which could be hosted on a personal computer (PC), to assist in researching the dynamic behaviour of materials during fast crack propagation. A model was required which could be easily reconfigured to represent different materials, provide good visibility of the dynamic fracture processes being simulated and generate information to complement the results of experimental research. A version of the devised PC model and its use with a recently developed experimental technique to achieve rapid crack propagation in a small specimen, is presented.

1. Introduction

Powerful fracture models that can simulate in great detail crack initiation and propagation processes in a wide variety of materials are very useful. They do, however, require complex and very refined finite element (FE) or other programs and these need to be hosted on powerful, expensive computers. The running costs of these models are high and they can require considerable expertise, time and effort to reconfigure for different studies. There are, however, a variety of less powerful modelling methods which, for example, use a coarser FE mesh and fill in the spaces, where necessary, with approximating subroutines. These can be more economic to use but still require more processing power than is available from PCs.

A general difficulty with modelling is to achieve a realistic opening of the crack tip and there are a variety of subroutines used to improve this feature of models. The problem with an ordered, spaced array of elements is that as bonds along the crack path are released or broken, the model's structure can be excited into complex transient oscillations and these will vary depending upon crack velocity and other factors. A subroutine or other method is needed to constrain the opening of the crack tip to be more realistic, or at least smooth out the effect of step functions generated along the crack path. One such subroutine, frequently used in FE models is a constraining hold-back force which is applied to elements along the crack path as their bonds are broken [1–6].

PCs are frequently used in fracture laboratories to control experimental equipment and for data collection so it would be very convenient to have a PC fracture model. This is particularly so when there are advantages in combining modelling and experimental facilities. It is important that the PC model should be easy to reconfigure for different fracture studies. For some research, more powerful fracture models and computers are needed so it helps if the PC model and all other models in the laboratory can provide data in the same format. The objective, therefore, was to produce a simple, general-purpose, fracture PC modelling tool which would be easy to modify and provide good visibility of the simulated fracture processes.

A version of the devised PC model is related to experimental results obtained from the use of small, tongue specimens to research fast fracture in viscoelastic materials [7]. The tongue extended the intended crack path and was used for launching a fast crack into the main section of the specimen which was strained but in a stress relaxed state. Another reason for using the tongue technique was that the crack upon arriving in the main section of the specimen was already of considerable effective length which reduced crack launching problems.

2. PC Model

The distributed mass and spring network as illustrated in Fig. 1 was found to be suitable for the PC model. Only one half of the single edge-notch specimen was modelled as shown by the distributed array of element masses. Lines between the masses represent A springs in the vertical direction and B springs in the horizontal direction as annotated in Fig. 1. Viscous damping can be added when required to these elements and across the crack tip. Other mass, spring, viscous damping networks can be used but the simple option shown was well able to provide data which compared closely with those obtained from more powerful FE and other models. An advantage of these distributed mass, spring, viscous damping networks is that they fit neatly between Williams' [8, 9] lumped constant models and powerful dynamic FE models [1–6].

When the PC model is tensioned normal to the crack path, strain energy is initially stored in the A springs and the B springs are unaffected. When the

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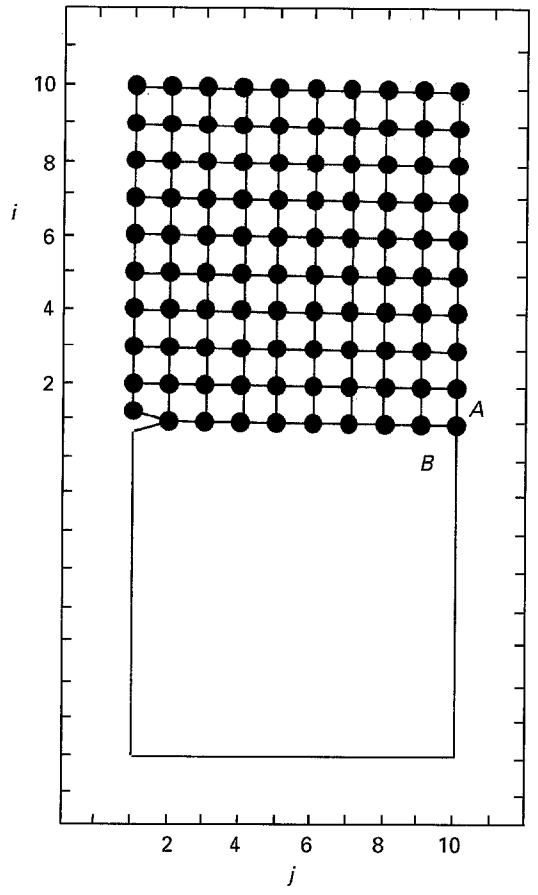

Figure 1 Distributed mass, spring network showing initial displacement of masses, A and B spring elements. (Mass displacements x_{ij} ($i = 1, 10$ and $j = 1, 10$)).

crack is propagating, masses along the designated crack path are released and the A springs above the opening crack start to relax and their strain is shared with the B springs. PC machines are well able to do step-by-step integration to represent this dynamic strain transfer process. This is the process resulting in a build up of shear strain at and ahead of the crack tip. The rate and extent of the redistribution and build up of strain at the crack tip is thus determined by the response characteristics of the distributed mass, spring network. It follows that the faster the crack bonds are broken, the less time there is for the network to effect the conversion of tensile to shear strain energy and transfer it to the advancing crack tip. The Poisson's ratio of the material to be represented is used to determine the relative stiffness of the A and B springs. Similarly, the PC model's mass, distributed viscous damping and energy absorbing elements at the crack tip are related to the material to be simulated.

An advantage of the mass, spring, viscous damping network is the simplicity of the step-by-step integration that is needed to represent the dynamic behaviour of the structure. These mathematical processes are well within the capability of most PCs. In essence, all that is needed are equations of motion for each of the masses in the modelling network being used. These equations provide a basis for a step-by-step program to compute the motion of the sprung masses as the crack moves along the designated path. For the model shown in Fig. 1, these equations are of the standard form

$$\begin{aligned} ma_{ij} = & -k_A(x_{ij} - x_{i+1\,j}) - k_A(x_{ij} - x_{i-1\,j}) \\ & - k_B(x_{ij} - x_{ij+1}) - k_B(x_{ij} - x_{ij-1}) \\ & - \eta_A(v_{ij} - v_{i+1\,j}) - \eta_A(v_{ij} - v_{i-1\,j}) \\ & - \eta_B(v_{ij} - v_{ij+1}) - \eta_B(v_{ij} - v_{ij-1}) \quad (1) \end{aligned}$$

which is a function of displacement (x_{ij}), velocity (v_{ij}) and acceleration (a_{ij}) of each mass (m) where the stiffness and damping of the A and B springs are k_A, η_A and k_B, η_B respectively. Each element in the model would need to relate to the size and properties of a corresponding element in a real material and the following factors need to be taken into account

$$m = \rho BL^2 \quad (2)$$

$$k_A = EB, \quad k_B = EB/2(1 + \nu) \quad (3)$$

$$\eta_A = 2\xi_A(k_A m)^{1/2}, \quad \eta_B = 2\xi_B(k_B m)^{1/2} \quad (4)$$

where ρ is density, E is modulus, ν is Poisson's ratio, B is thickness, L is spacing of masses and ξ_A and ξ_B are non-dimensional damping ratios for A and B springs. When making specific comparisons between different materials but identically shaped specimens then the equations can be simplified by careful normalization. Later in this paper, such a comparison is made between experimental and PC model data for three different polymers. A modelling point is that care is needed in selecting a suitably small time-step to reduce the build up of computational error and particularly so when the transient behaviour of the structure is complex.

The crack tip conditions can be modelled in several ways but the version used here was the same as that often employed in FE networks namely a hold-back force [1, 2]. The reason for this was that in this study, it was required to generate data from the PC model that could be directly compared with those from more powerful FE models. The PC model as with most FE models needed a subroutine to fill in the gaps between the spaced out bonds along the crack path. This was the function of the hold-back force which was also used to control the work done at the crack tip. The arrangement was that as an element's bond at the crack tip was broken, the hold-back force was applied to constrain the opening movement of the released mass. A fast removal of the hold-back force would result in the model behaving like a brittle material whereas a slow removal of this force would result in more work being done at the crack tip as would be the case for viscoelastic materials. For these studies, then at any time, the hold-back force was only applied to the mass directly above the last bond broken along the crack path but there are other options that can be used.

The height to width ratio of the PC model can be changed to study geometric effects on crack propagation in materials. When simulating a fixed grip configuration, then, as the height of the specimen is reduced, the reflections of wavefronts from the grips will modify the strain pattern around the crack. This would have considerable effect on slow cracks but less effect as the

crack advances more quickly. For very fast cracks approaching the limiting velocity, C_L, then the crack tip will be little aware of reflected wavefronts which will act on the crack well behind the crack tip. The effective height of the PC model will, of course, generally limit the strain energy available to open the crack. All these conditions can be studied using the simple PC model.

In this study, the PC model is used in two different operational modes which are the same as those often used in FE models. One is to strain the PC model in tension normal to the intended crack path and sequentially release bonds along the crack path at different rates, which is sometimes referred to as the generation mode. This is to monitor the dynamic behaviour of the PC model in transferring strain energy to the crack tip. The rate of work done at the crack tip is determined by the hold-back force which constrains the opening of the crack tip. For the generation mode, this hold-back force decays linearly at a rate determined by the crack velocity. The other operational mode is to provide for the crack to be self-propagating.

Running the PC model in propagation mode presents a close to realistic fast crack behaviour in actual materials. The main difference between this and the generation mode is that the hold-back force arrangement needs to be changed and a condition for breaking of the crack path bonds needs to be programmed into the model. In the case of the hold-back force, this is simply a matter of decreasing the hold-back force as the crack tip opens instead of relating it to crack tip velocity [2]. A bond-breaking level is introduced and is one which the dynamic strain at the crack tip must exceed. It follows that for the same applied tensile strain, then, the lower the bond breaking level, the lower its threshold load, p_0, to just propagate a crack. Not to be overlooked is that local yielding at the crack tip will increase the value of p_0. Similarly, the generation of fibrils across the crack path, energy absorption by micro-cracks about the main crack, the generation of a hot-spot at the crack tip and other crack tip propagation effects modify the bond-breaking conditions at the crack tip. These crack tip effects can be simulated by the model by changing the hold-back force characteristic. In this study, a simple linear rate of decreasing the hold-back force was used.

3. Experimental procedure

The use of the PC fracture model devised for this study was to complement experimental research in the analysis of crack propagation in viscoelastic materials. The tongue specimen experimental technique [7] was developed to achieve rapid crack propagation in small specimens under steady applied load conditions. It was a requirement that these specimens should be strained, allowed to stress relax and a crack initiated normal to the applied strain without disturbing the established steady state stress conditions.

A tongue was used to extend the designated crack path as shown in Fig. 2. For tough materials, such as polyethylene, the tongue was freeze-cooled, using liquid nitrogen, until it could be easily fractured using a shallow three-point bend device. This is also shown in Fig. 2 together with instrumentation sensors adjacent to the crack path to monitor the rise and fall of strain about the passing crack tip. Crack velocity was also measured by these sensors as well as by the breaking of conducting strips at the beginning and end of the crack path. A variety of other tongue fracture devices were used, other than the three-point bend one, to achieve the launching of sharp, fast cracks into the main section of specimens of different materials. Only the first 10 mm of the crack path in the main section of the specimen could be affected by the presence of the tongue. This was to include any distortion of strain in the main section near to the root of the tongue, effects of cooling the tongue and applying fracture forces to the tongue. These conditions were fully satisfied by the experimental method [7]. An additional point was that on leaving the first 10 mm of the crack path, in the main section of the specimen, the crack should have sufficient effective length to achieve quickly near to constant velocity through the remaining length of the specimen when this was appropriate. This was another reason for extending the crack path of the specimen with a tongue section.

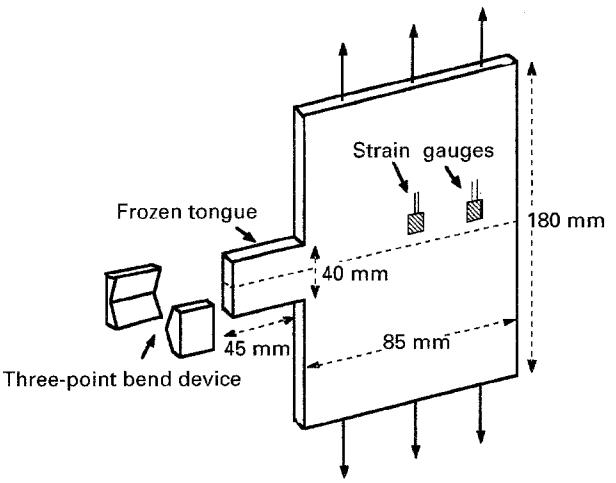

Figure 2 Schematic of tongue geometry with three-point bend device shown.

4. Results

Fig. 3 shows two outputs when the PC model is in the generation mode when the crack is not self-propagating but is represented by a sequential release of bonds along the crack path. The generation mode crack velocity in Fig. 3(a) is $0.2C_s$ and in Fig. 3(b) is $0.8C_s$ where C_s is the shear wave velocity in the distributed mass, spring network. These two figures show the variation with crack growth of total strain energy, total kinetic energy and work done against the hold-back force normalized with respect to the initial strain energy given to the model by the external applied force. This shows that for a rapidly propagating crack, a large part of the total strain energy about the opening crack is being used to accelerate the structure's masses and this limits that available to the crack tip

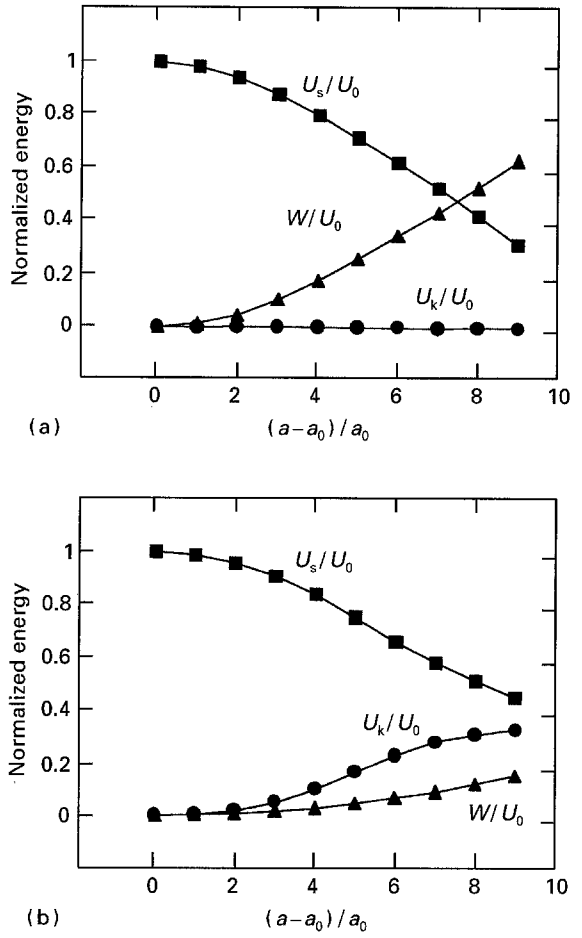

Figure 3 Normalized plots of total strain energy (U_s/U_0), total kinetic energy (U_k/U_0) and work done against hold-back force (W/U_0) where U_0 is the initial strain energy versus normalized crack extension ($(a - a_0)/a_0$) where a_0 is the initial crack length. This is for crack velocities of (a) $0.2C_s$ and (b) $0.8C_s$ where $C_s = L(k_B/m)^{1/2}$ is the shear wave velocity in the PC model.

(Fig. 3(b)). For a slowly propagating crack, a larger proportion of the total strain energy is available to the crack tip because of the demand for kinetic energy by the masses is minimal (Fig. 3(a)). The dynamic energy release rate, G^{dyn}, which is equal to the crack resistance, R, is given by

$$G^{dyn} B\Delta a = \Delta U_e - \Delta U_s - \Delta U_k - \Delta U_d \quad (5)$$

where B is thickness, Δa is the increment of crack growth, ΔU_e is the external work done, ΔU_s is the change of strain energy, ΔU_k is the change of kinetic energy and ΔU_d is the dissipated energy in the distributed viscous damping elements. G^{dyn} can also be obtained from the PC model as the work done against the hold-back force which resists the opening of the crack.

Fig. 4 shows how G^{dyn} increases along the crack path for four different simulated crack velocities from $0.2C_s$ up to $0.8C_s$. This also illustrates well that the dynamic response of the network is less able to deliver strain energy to the crack tip the higher its velocity. In a very large specimen, G^{dyn} will continue to increase with crack length but as the height of the specimen is reduced, this will impose a limit on the increase of G^{dyn} and a plateau will be reached. This specimen height

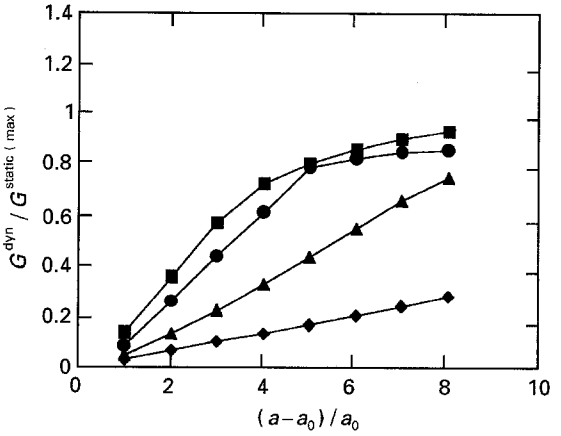

Figure 4 G^{dyn} normalized with respect to $G^{static(max)}$ versus normalized crack extension ($(a - a_0)/a_0$). This is for crack velocities of: $0.2C_s$ (■), $0.4C_s$ (●), $0.6C_s$ (▲) and $0.8C_s$ (◆). (For a given height of specimen in the PC model, $G^{static\,(max)}$ is the maximum value of G^{static} attainable).

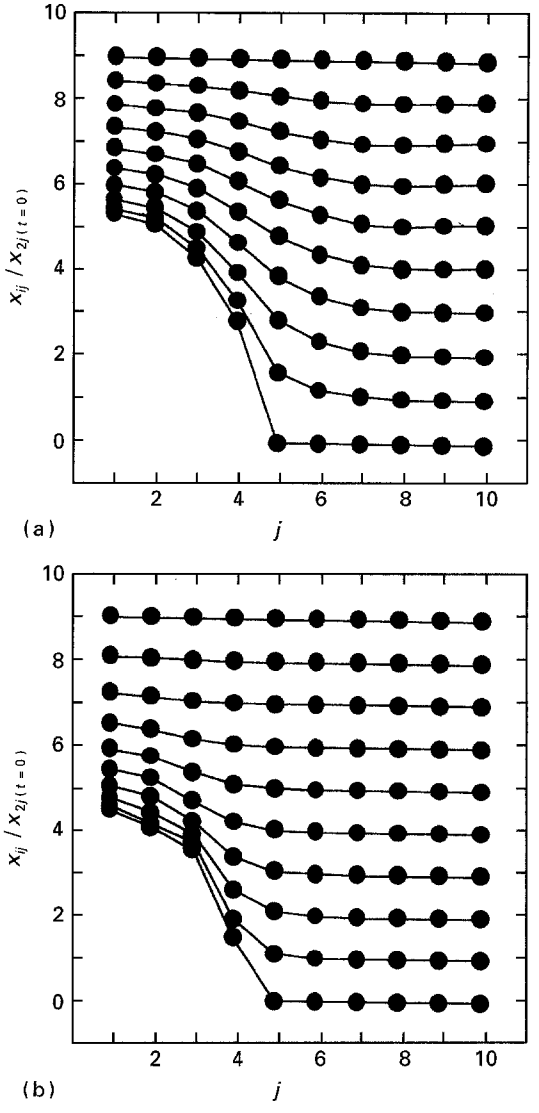

Figure 5 Normalized displacements ($x_{ij}/x_{2j(t=0)}$) of the model's elements perpendicular to the crack path when $a/a_0 = 5$ for crack velocities of (a) $0.2C_s$ and (b) $0.8C_s$.

limiting effect on G^{dyn} is first noticeable for slow crack velocities, as evident in Fig. 4.

Fig. 5 shows the height effect in a different way, namely that for the same crack length, the relaxation

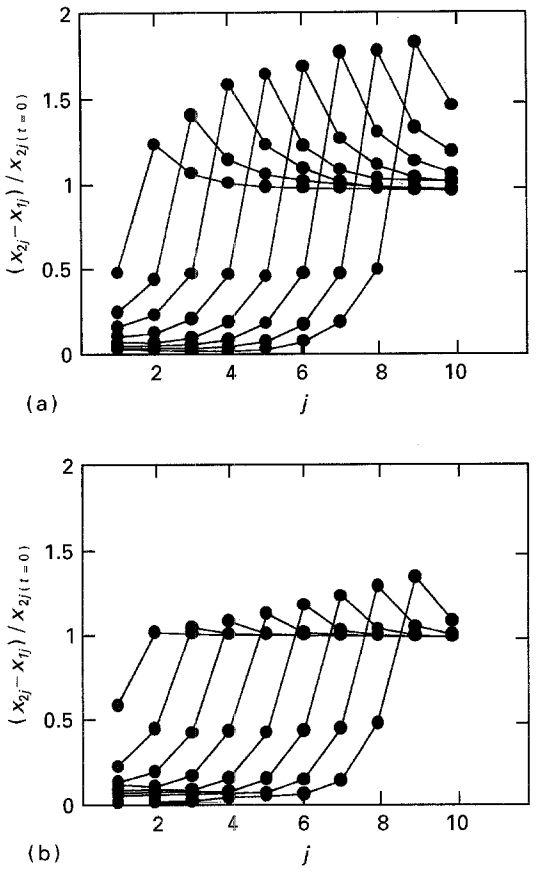

Figure 6 Normalized extension of A elements $((x_{2j} - x_{1j})/x_{2j(t=0)})$ versus j to show the increase of strain at the crack tip as each bond is broken. This is for crack velocities of (a) $0.2C_s$ and (b) $0.8C_s$.

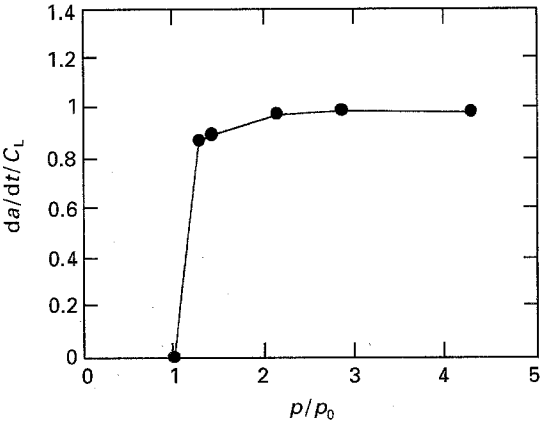

Figure 7 Normalized crack velocity ($da/dt/C_L$) versus normalized applied load (p/p_0).

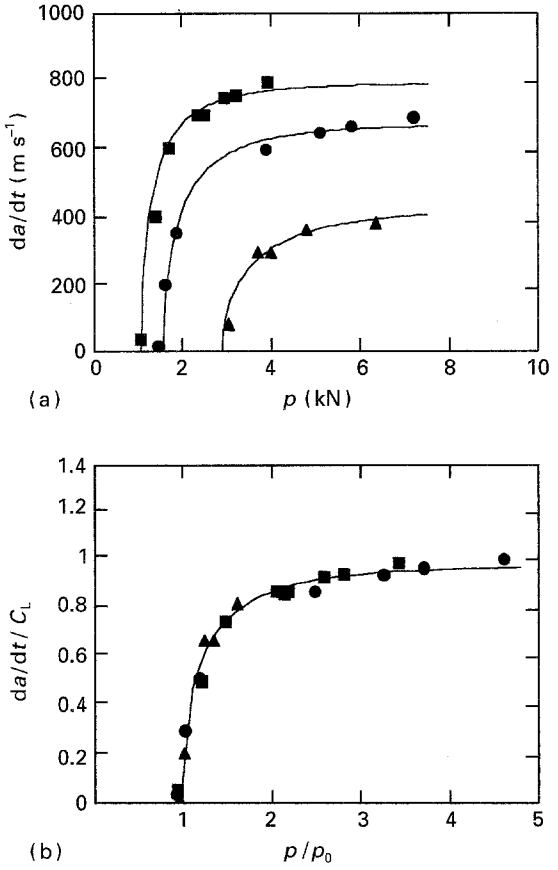

Figure 8 Experimental data from tongue technique. (a) Crack velocity (da/dt) versus stress relaxed load (p) for polymethylmethacrylate (■), polycarbonate (●) and polyethylene (▲); (b) normalized crack velocity ($da/dt/C_L$) versus normalized applied load (p/p_0) for the same materials.

of the model's elements has reached the upper layer of nodes in the case of the slow crack ($0.2C_s$ – Fig. 5(a)) but not in the case of the fast crack ($0.8C_s$ – Fig. 5(b)). For clarity in Fig. 5, the vertical spacing of the masses relates only to the extension of the model's A elements normal to the crack path. The horizontal displacement of the masses relates to their physical separation along the crack path.

The dynamic build up of strain at the crack tip is shown in Fig. 6, for two crack velocities ($0.2C_s$ and $0.8C_s$), which then falls to zero behind the crack tip for each of the bonds broken. For slow rates of breaking bonds (Fig. 6(a)), the model's A springs have had considerable time to relax and transfer strain to the B springs and so increase the build up of dynamic strain at the crack tip whereas for rapid breaking of the bonds (Fig. 6(b)), the build up of strain at the crack tip is small. These curves simulate well the data obtained from strain gauges adjacent to the crack path in experimental studies.

Fig. 7 shows crack velocity versus applied load obtained from running the PC model in a propagation mode. The crack velocity is normalized with respect to the limiting crack velocity condition, C_L, and the load is normalized with respect to the threshold load condition, p_0. This curve is compared with that obtained by experiment.

From experimental research using the tongue specimen technique, Fig. 8(a) shows crack velocity versus stress relaxed load curves for three materials: polymethylmethacrylate, polycarbonate and polyethylene. These data are normalized in Fig. 8(b) with respect to limiting crack velocity, C_L, and threshold load, p_0, for each material. Fig. 8(b) is directly comparable with the PC model data shown in Fig. 7, with which there is good agreement. The generation mode data obtained from the PC model and shown in Figs 3–6 show the dynamic processes which in actual materials generate strain at the crack tip. These dynamic process data are less easy to obtain directly by experiment.

5. Discussion

A summary of the main features of the distributed mass, spring, viscous damping model are listed below.

1. The structure of the PC model is easy to program and host on a PC.
2. The PC model is easy to change to represent the different properties of materials e.g. stiffness, density, Poisson's ratio, viscous damping and energy absorbing processes at the crack tip.
3. Good visibility is provided by the PC model's behaviour of the dynamic fracture processes it is simulating.
4. The PC model can provide data in the same format as that generated by more powerful FE models.
5. The PC modelling program can be written in simple "BASIC", "C" or other faster to run and more versatile languages.

The use of the PC model to assist in the analysis of results from the tongue specimen experimental technique for researching fast fracture highlighted the following points.

1. The sharp crack initiated in the tongue of the specimen arrived in the main section with sufficient effective length for the crack propagation to continue at nearly constant velocity through the specimen when appropriate.
2. The threshold load to just provide for a crack to propagate through the main section of the specimen was that to overcome the strength of the bonds along the designated crack path and local yielding at the crack tip.
3. Local yielding at the crack tip increased the value of the threshold load needed.
4. As the height of the specimen is reduced, there comes a point when the reflections from the fixed grips tensioning the specimen can modify the strain pattern around the crack tip. Also, the reduced height of the specimen will generally limit the strain energy available to the crack tip.
5. The faster the crack propagates then the lower the specimen height can be before the above effects are noticeable. Very fast cracks rely mostly on strain energy available local to the crack tip and are little aware of wave reflection effects. These very fast cracks tend to have short effective lengths which do not vary greatly so they tend to travel at near to constant velocity as they approach the limiting condition, C_L.

It is important to validate all modelling methods to assess their suitability for the purpose for which they are to be used. An advantage is if PC models, FE models and others used in a laboratory are able to produce data in the same format as this much helps in validation and verification assessments. Good visibility of modelling processes and how these relate to those in materials being studied is very helpful for many research activities. A big advantage of computer models of fracture is the ability and ease to generate data which is difficult or impossible to obtain by experiment.

6. Conclusions

A useful PC fracture model has been devised which is compatible with FE and other more powerful models in that it can provide data in a similar format. The distributed mass, spring, viscous damping PC model is able to function in both generation and propagation modes and the quality of the information produced is not dramatically less than that from more powerful models. An advantage of the PC model is that it is easy to program in several of the popular computer languages and can be hosted on PCs used in many fracture laboratories for controlling experimental equipment and data collection. Overall, it is thought the PC fracture model is a good general purpose tool that can be easily programmed and used at low cost.

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